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Real-time dynamics in the 1+1 D abelian Higgs model with fermions*

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In approximate dynamical equations, inhomogenous classical (mean) gauge and Higgs fields are coupled to quantized fermions. The equations are solved numerically on a spacetime lattice. The fermions appear to equilibrate according to the Fermi-Dirac distribution with time-dependent temperature and chemical potential.

1. The real-time path integral for quantum fields is very difficult to evaluate numerically and approximations need to be made before giving the problem to the computer. Two types of approximations are currently in use: classical and gaussian, such as Hartree, large N. The classical approximation gives valuable nonperturbative results [1] but suffers complications due to (Rayleigh-Jeans type) divergencies. The gaussian approximation has the benefit of staying within the quantum domain where we know how to deal with divergencies, but it is not good enough for large times. Naturally, one would like to combine the good aspects of both approximations [2]. A crucial test is to see whether the system equilibrates quantum-like, and not classical equipartition-like, despite the fact that one is just solving a large number of coupled nonlinear equations which conserve energy. This is one motivation for the present study. Another is the intrinsic interest in the complicated nonperturbative dynamics of the abelian Higgs model with fermions.

2. The 1+1 D abelian Higgs model coupled axially to fermions is qualitatively similar to the electroweak sector of the Standard Model. As for the SU(2) case in 4D, it can be rewritten in a form with vectorial gauge couplings and Majorana-Yukawa couplings. For $N \to \infty$ fermion replicas, the equations of motion reduce to a classical field approximation for the bosonic variables, with a quantal fermion backreaction [3]:

$$\begin{aligned}\partial_\mu F^{\mu\nu} + e^2 i(D^\nu\varphi^*\,\varphi - \varphi^* D^\nu\varphi) & \\ + (e^2/2)\langle\bar\psi i\gamma^\nu\psi\rangle &= 0,\\ (-D_\mu D^\mu + \mu^2 + 2\lambda\varphi^*\varphi)\varphi &= 0,\end{aligned}$$

where we have specialized to zero Yukawa coupling. The fermion backreaction is specified as follows. Introduce a complete set of orthonormal mode functions u_α, v_α for the fermions, which satisfy the Dirac equation

$$\gamma^\mu D_\mu u_\alpha = 0, \quad \gamma^\mu D_\mu v_\alpha = 0.$$

Next define the fermion operator $\hat\psi$,

$$\hat\psi(x) = \sum_\alpha [\hat b_\alpha u_\alpha(x) + \hat d_\alpha^\dagger v_\alpha(x)],$$

in terms of annihilation and creation operators $\hat b_\alpha$, $\hat b_\alpha^\dagger$, ... The fermion back reaction is then specified by the initial conditions $\langle \hat b_\alpha^\dagger \hat b_{\alpha'}\rangle = n_\alpha\,\delta_{\alpha\alpha'}$, $\langle \hat d_\alpha^\dagger \hat d_{\alpha'}\rangle = \bar n_\alpha\,\delta_{\alpha\alpha'}$, etc.

The above system of equations has been implemented on a lattice using Wilson's fermion method for the spatial derivative and the staggered fermion interpretation for a 'naive' discrete time derivative [3]. Usual expectations on fermion number non-conservation tied to sphaleron transitions are correctly represented on the lattice. For simplicity we continue with continuum notation.

3. Here we are especially interested in thermalization properties of the fermions. We tested

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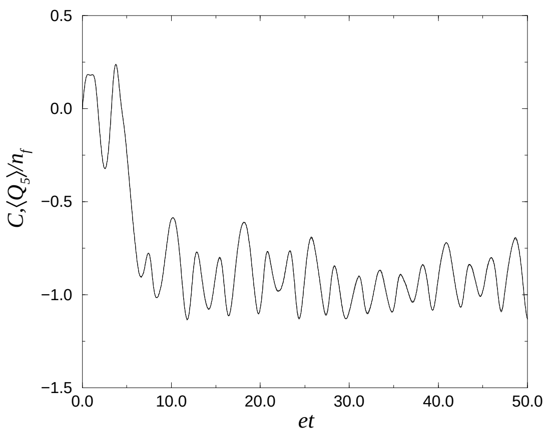

Figure 1. Chern-Simons number C and axial charge $\langle Q_5\rangle/n_f$ versus time in units $1/e$.

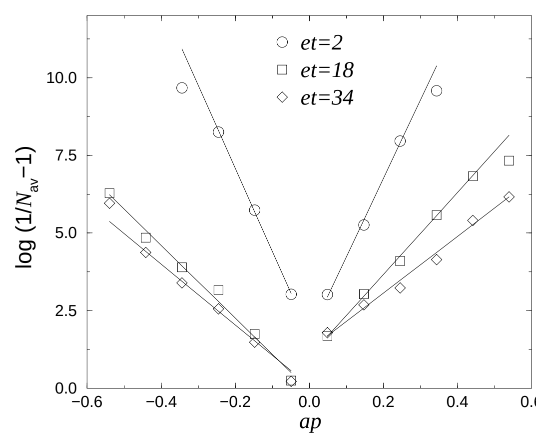

Figure 2. Least squares straight line fits to $\ln(N_{\mathrm{av}p}^{-1} - 1)$ versus ap.

[4] for local (in time) equilibration of fermions by comparing their distribution function with the Fermi-Dirac distribution. The distribution function was identified from the equal-time fermion two point function, averaged over space (a circle with circumference L),

$$S(z,t) = \frac{1}{L}\int_0^L dx\, \langle\psi(x,t)\bar{\psi}(x+z,t)\rangle_{\mathrm{g.f.}}.$$

Here g.f. indicates a complete gauge fixing. Alternatively, the two point function can be rendered gauge invariant by supplying a parallel transporter $U(x,y) = \exp[-i\int_x^y dz\, A_1(z)/2]$. We have used the latter method, but it is actually closely related to complete Coulomb gauge fixing [4]. If the fermions were free, the Fourier transform

$$S(p,t) = \int_0^L dz\, e^{-ipz}\, S(z,t)$$

would be given in terms of distribution functions N_p, $\bar{N}_p$ als follows:

$$\begin{aligned}
\mathrm{Tr}\, S(p,t) &= [1 - N_p(t) - \bar{N}_{-p}(t)]\,\frac{m_p(t)}{\omega_p(t)},\\
\mathrm{Tr}\, i\gamma^1 S(p,t) &= [1 - N_p(t) - \bar{N}_{-p}(t)]\,\frac{p}{\omega_p(t)},\\
\mathrm{Tr}\, i\gamma^0 S(p,t) &= 1 - N_p(t) + \bar{N}_{-p}(t),\\
\omega_p(t) &= \sqrt{m_p^2(t) + p^2},
\end{aligned}$$

with $\mathrm{Tr}\,\gamma_5 S(p,t) = 0$ because of parity invariance. For free fermions N_p, $\bar{N}_p$ and m are time-independent. Assuming that the interacting model can be described approximately by quasi-particles, we now use the above equations to *define* $N_p(t)$, $\bar{N}_p(t)$ and $m_p(t)$. In a non-equilibrium situation they will depend on time.

The simulations had the following parameters: $n_f = 2$ flavors (related to fermion doubling in time), spatial size $m_\varphi L \approx 6.4$, $\lambda/e^2 = 0.25$ ($m_A L \approx 9$), coupling $e^2/m_\varphi^2 \approx 0.25$, with spatial lattice spacing $am_\varphi \approx 0.10$, temporal spacing $a_0/a = 0.005$ and $L/a = 64$ spatial lattice sites.

Fig. 1 shows the Chern-Simons number $C = -\int dx\, A_1/2\pi$ and the axial charge $\langle Q_5\rangle/n_f$ for a simulation starting with a fermionic vacuum (i.e. $n_\alpha, \bar{n}_\alpha = 0$) and some kinetic energy stored in a few low momentum modes of the Higgs field. The anomalous fermion number non-conservation equation $\Delta\langle Q_5\rangle = n_f \Delta C$, is well obeyed since the two curves are indistinguishable (initially $\langle Q_5\rangle = C = 0$). The oscillations correspond roughly to the basic period $2\pi/m_A$. To smoothen these we average $S(p,t)$ over a time interval t_{av} before extracting the distribution functions. We used $et_{\mathrm{av}} = 4$ and studied the behavior of $N_{\mathrm{av}p}(t) \equiv$

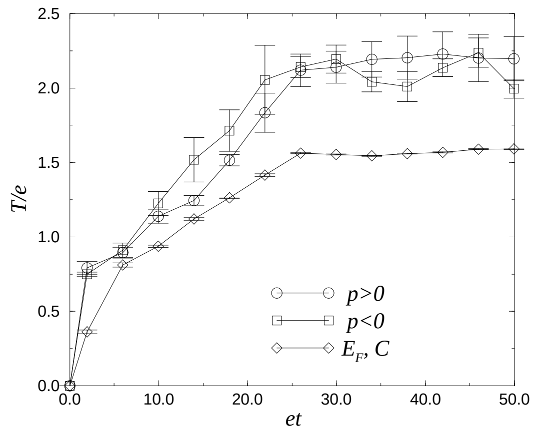

Figure 3. Effective temperatures $T(t) = 1/\beta(t)$ versus time obtained from fits as in figure 2.

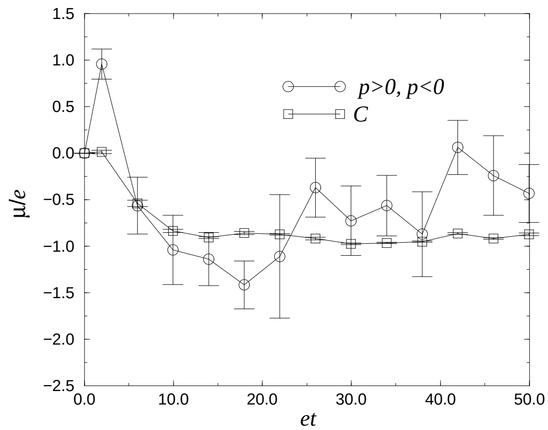

Figure 4. As in figure 3 for the effective chemical potential.

$[N_p(t) + \bar{N}_{-p}(t)]/2$. As expected, $m_p(t)$ is effectively zero. Surprisingly, $N_{\mathrm{av}p}(t)$ resembles quite fast a Fermi-Dirac distribution for inverse temperature β and Q_5-chemical potential μ:

$$
\begin{aligned}
f_p(\beta,\mu) &= \{\exp[\beta(E_p - \mu q_{5p})] + 1\}^{-1}, \\
E_p &= |p|, \quad q_{5p} = p/|p|
\end{aligned}
$$

(the axial charge of a fermion depends on the sign of p). Fig. 2 shows $\ln(N_{\mathrm{av}p}^{-1} - 1)$ versus ap at various times. We see linear behavior, $\ln(N_{\mathrm{av}p}^{-1} - 1) \approx \beta(t)[|p| \pm \mu(t)]$, suggesting local (in time) equilibrium. The distribution functions of modes with $ap \geq 0.5$ are consistent with zero, so these modes are practically not excited. Furthermore, since the relevant p are small in lattice units, discretization effects are reasonably small. To achieve this the initial energy stored in the Bose fields has to be in the relatively low momentum modes only – the fields are far from classical equilibrium.

Figs. 3 and 4 show the effective temperature and chemical potential as a function of time. Note that $T_{p>0}(t) \approx T_{p<0}(t)$. If the fermions were free, then their energy and axial charge densities would follow from the Fermi-Dirac distribution according to

$$
\begin{aligned}
E_F/L &= n_f(\pi T^2/6 + \mu^2/2\pi), \\
\langle Q_5\rangle/L &= n_f\mu/\pi.
\end{aligned}
$$

Conversely, E_F and $\langle Q_5\rangle$ imply an effective temperature and chemical potential; these are also plotted in Figs. 3,4 (data labeled E_F, C resp. C). The E_F, C-temperature appears systematically lower that that from N_{avp}. This may be due to the fact that the fermions are not free.

4. In conclusion, we see evidence for fast equilibration of fermions coupled to classical Bose fields. It is important that the classical fields can be spatially inhomogeous, since this allows the fermions to scatter nontrivially, e.g. by their (screened) Coulomb interaction. The Bose fields have created fermions and lost some energy, and they are not in (classical) equilibrium in the time span shown here. More details on this work can be found in [4].

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